

BINOMIAL THEOREM

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 C

$$\begin{aligned}
 (1+x)^n &= \sum_{r=0}^n {}^nC_r x^r = ({}^nC_0 x^0 + {}^nC_2 x^2 + \dots) \\
 &+ ({}^nC_1 x^1 + {}^nC_3 x^3 + \dots) \\
 &= (b+a) \quad \begin{cases} T_2 + T_4 + T_6 + \dots = b \\ T_1 + T_3 + T_5 + \dots = a \end{cases} \\
 \therefore (1-x)^n &= ({}^nC_0 x^0 + {}^nC_2 x^2 + \dots) \\
 &- ({}^nC_1 x^1 + {}^nC_3 x^3 + \dots) \\
 &= b - a \\
 (1-x^2)^n &= (1+x)^n (1-x)^n = (a+b)(a-b) \\
 &= b^2 - a^2
 \end{aligned}$$

Sol.2 C

$$\begin{aligned}
 &\left(x^{\frac{1}{3}} - x^{-\frac{1}{2}}\right)^{15} \\
 T_{r+1} &= {}^{15}C_r \left(x^{\frac{1}{3}}\right)^{15-r} \left(-x^{-\frac{1}{2}}\right)^r \\
 T_{r+1} &= {}^{15}C_r (-1)^r (x)^{\frac{15-r}{3} - \frac{r}{2}} = {}^{15}C_r (-1)^r (x)^{\frac{30-5r}{6}} \\
 \text{Given if } \frac{30-5r}{6} &= 0 \text{ then } T_{r+1} = 5m, m \in \mathbb{N} \\
 \Rightarrow r &= 6 \Rightarrow T_7 = 5m \\
 T_7 &= {}^{15}C_6 (-1)^6 = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2} \\
 &= 5 \cdot 7 \cdot 13 \cdot 11 = 5 \cdot (1001) \Rightarrow m = 1001
 \end{aligned}$$

Sol.3 B

$$\begin{aligned}
 7^{\text{th}} \text{ term of } \left(2^{\frac{1}{3}} + 3^{-\frac{1}{3}}\right)^n &\text{ is } T_7 = {}^nC_6 2^{\frac{n-6}{3}} 3^{-2} \\
 \text{From beginning} \\
 7^{\text{th}} \text{ term of } \left(2^{\frac{1}{3}} + 3^{-\frac{1}{3}}\right)^n &\text{ from the end} \\
 &= 7^{\text{th}} \text{ term of } \left(3^{-\frac{1}{3}} + 2^{\frac{1}{3}}\right)^n \text{ from the beginning} \\
 &= T'_7 = {}^nC_6 3^{-\frac{(n-6)}{3}} 2^2
 \end{aligned}$$

$$\text{given that } \frac{T_7}{T'_7} = \frac{1}{6}$$

$$\begin{aligned}
 \Rightarrow \frac{{}^nC_6 2^{\frac{n-6}{3}} 3^{-2}}{{}^nC_6 3^{-\frac{(n-6)}{3}} 2^2} &= \frac{1}{6} \Rightarrow \frac{2^{\frac{n-6}{3}-2}}{3^{2-\frac{(n-6)}{3}}} = \frac{1}{6} \\
 \Rightarrow \frac{1}{(2 \cdot 3)^{2-\frac{(n-6)}{3}}} &= \frac{1}{6} \Rightarrow 2 - \frac{n-6}{3} = 1 \\
 \Rightarrow 3 &= n-6 \Rightarrow n = 9
 \end{aligned}$$

Sol.4 A

$$\begin{aligned}
 \left(1 + \frac{1}{4n}\right)^{4n} &= {}^{4n}C_0 \left(\frac{1}{4n}\right)^1 + {}^{4n}C_2 \left(\frac{1}{4n}\right)^2 + \dots \\
 &+ {}^{4n}C_{4n} \left(\frac{1}{4n}\right)^{4n} \\
 &= 1 + 1 + \frac{1}{2!} \left(1 + \frac{1}{4n}\right) + \dots + \frac{1}{4n!} \left(1 - \frac{1}{4n}\right) \left(1 - \frac{2}{4n}\right) \dots \left(1 - \frac{4n-1}{4n}\right) \\
 &< 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(4n)!} \\
 \left(1 + \frac{1}{3n}\right)^{3n} &= 1 + 1 + \frac{1}{2!} \left(1 + \frac{1}{3n}\right) + \dots \\
 &+ \frac{1}{(3n)!} \left(1 - \frac{1}{3n}\right) \left(1 - \frac{2}{3n}\right) \dots \left(1 - \frac{3n-1}{3n}\right) \\
 &< 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(3n)!} \\
 \left(1 + \frac{1}{2n}\right)^{2n} &= 1 + 1 + \frac{1}{2!} \left(1 + \frac{1}{2n}\right) + \dots \\
 &+ \frac{1}{(2n)!} \left(1 - \frac{1}{2n}\right) \left(1 - \frac{2}{2n}\right) \dots \left(1 - \frac{2n-1}{2n}\right) \\
 &< 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(2n)!}
 \end{aligned}$$

$$\left(1 + \frac{1}{n}\right)^n = 1 + 1 + \frac{1}{2!} \left(\frac{1}{n}\right) + \dots$$

$$+ \frac{1}{(n)!} \left(\frac{1}{n}\right) \left(\frac{1}{n}\right) \dots \left(\frac{1}{n}\right)$$

$$< 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(2n)!}$$

$$\left(1 + \frac{1}{n}\right)^n < \left(1 + \frac{1}{2n}\right)^{2n} < \left(1 + \frac{1}{3n}\right)^{3n} < \left(1 + \frac{1}{4n}\right)^{4n}$$

Sol.5 C

$$T_{r+1} = {}^nC_r (2)^{n-r} \left(\frac{x}{3}\right)^r = {}^nC_r \frac{2^{n-r}}{3^r} x^r$$

$$\text{Coeff } T_8 = T_9 \Rightarrow {}^nC_7 \frac{2^{n-7}}{3^7} = {}^nC_8 \frac{2^{n-8}}{3^8}$$

$$\Rightarrow \frac{{}^nC_8}{{}^nC_7} = \frac{3 \cdot 2}{1} \Rightarrow \frac{n-8+1}{8} = 6$$

$$\Rightarrow n-7 = 48 \Rightarrow n = 55$$

Sol.6 D

$$\frac{1}{\sqrt{4x+1}} \left[\left(\frac{1+\sqrt{4x+1}}{2} \right)^7 - \left(\frac{1-\sqrt{4x+1}}{2} \right)^7 \right]$$

$$= \frac{1}{2^7 \sqrt{4x+1}} \left[\sum_{r=0}^7 {}^7C_r (1)^{7-r} (\sqrt{4x+1})^r - \sum_{r=0}^7 {}^7C_r (1)^{7-r} (-\sqrt{4x+1})^r \right]$$

$$= \frac{1}{2^7 \sqrt{4x+1}} [T_1 + T_2 + T_3 + \dots + T_8 - T_1 + T_2 - T_3 + \dots + T_8]$$

$$= \frac{1}{2^7 \sqrt{4x+1}} [2(T_2 + T_4 + T_6 + T_8)]$$

$$= \frac{1}{2^6 \sqrt{4x+1}} [{}^7C_1 \sqrt{4x+1} + {}^7C_3 (\sqrt{4x+1})^3$$

$$+ {}^7C_5 (\sqrt{4x+1})^5 + {}^7C_7 (\sqrt{4x+1})^7]$$

$$= \frac{1}{2^6} [{}^7C_1 + {}^7C_3 (4x+1)$$

$$+ {}^7C_5 (4x+1)^2 + {}^7C_7 (4x+1)^3]$$

= is the polynomial in x of degree 3

Sol.7 B

$$\text{G.T. is } T_{r+1} = {}^{100}C_r (2)^{\frac{100-r}{2}} (3)^{\frac{r}{4}}$$

The above term will be rational if exponent of 2 & 3 are integers.

$$\text{i.e. } \frac{100-r}{2} \text{ and } \frac{r}{4} \text{ must be integers}$$

the possible set of r is = {0, 4, 8, 16, ..., 100}
no. of rational terms is 26

Sol.8 B

If $n \in \mathbb{N}$ & n is even then

$$\frac{1}{1 \cdot (n-1)!} + \frac{1}{3! \cdot (n-3)!} + \frac{1}{5! \cdot (n-5)!} + \dots + \frac{1}{(n-1)! \cdot 1!}$$

$$= \frac{1}{n!} [{}^nC_1 + {}^nC_3 + {}^nC_5 + \dots + {}^nC_{n-1}]$$

n is even $\Rightarrow n-1$ is odd

${}^nC_{n-1}$ second Binomial coeff. from the end

$$= \frac{1}{n!} [C_1 + C_3 + C_5 + \dots + C_{n-1}]$$

$$= \frac{1}{n!} \cdot 2^{n-1} = \frac{2^{n-1}}{n!}$$

Sol.9 B

$$(1^2+1)1! + (2^2+1)2! + (3^2+1)3! + \dots + (n^2+1) \cdot n!$$

$$T_n = \sum_{n=1}^n (n^2+1)n!$$

$$= \sum_{n=1}^n [n^2 + 3n + 2 - 3n - 1]n!$$

$$= \sum_{n=1}^n [(n+2)(n+1) - 3(n+1) + 2]n!$$

$$= \sum_{n=1}^n [(n+2)! - 3(n+1)! + 2n!]$$

$$\begin{aligned}
 &= 3! + 4! + 5! + \dots + n! + (n+1)! + (n+2)! \\
 &\quad - 3[2! + 3! + 4! + 5! + \dots + n! + (n+1)!] \\
 &\quad + 2[1! + 2! + 3! + \dots + n!] \\
 &= [(n+2)! + (n+1)! - 3(n+1)! \\
 &\quad - 3 \cdot 2! + 2 \cdot 1! + 2 \cdot 2!] \\
 &= (n+2)! - 2(n+1)! - 2! + 2 \cdot 1! \\
 &= (n+1)! [n+2-2] \\
 &= (n+1)! \cdot n \\
 &= n \cdot (n+1)!
 \end{aligned}$$

Sol.10 D

$$\begin{aligned}
 3^{400} &= (3^2)^{200} = (9)^{200} = (10-1)^{200} \\
 &= {}^{200}C_0 10^{200} + {}^{200}C_1 10^{199} + {}^{200}C_2 10^{198} + \dots \\
 &\quad + {}^{200}C_{199} 10^1 + {}^{200}C_{200} 10^0 \\
 &= 100 \left[{}^{200}C_0 10^{198} + {}^{200}C_1 10^{197} + {}^{200}C_2 10^{196} + \dots \right. \\
 &\quad \left. + {}^{200}C_{198} \right] + {}^{200}C_{199} 10^1 + {}^{200}C_{200} 10^0 \\
 &= (k)100 + 200(10) + 1(1) \\
 &= (k+20)100 + 01 \\
 &\text{so last two digits is } 01
 \end{aligned}$$

Sol.11 A

$$\begin{aligned}
 C_0 + C_1 + C_2 + \dots + C_n &= 2^n = 256 \\
 \Rightarrow 2^n &= 2^8 \Rightarrow n = 8
 \end{aligned}$$

$$T_{r+1} = {}^8C_r (2x)^{8-r} \left(\frac{1}{x}\right)^r = {}^8C_r 2^{8-r} x^{8-2r}$$

$$\text{For Constant term} \Rightarrow 8 - 2r = 0 \Rightarrow r = 4$$

$$= {}^8C_4 2^4 = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2} 2^4 = 70 \times 16 = 1120$$

Sol.12 A

$$\text{sum of coeff of } (1 - 2x + 5x^2)^n = a$$

$$\text{sum of coeff of } (1 + x)^{2n} = b$$

$$\text{put } x = y = 1$$

$$a = (1 - 2 + 5)^n = 4^n \text{ \& } b = (1 + 1)^{2n} = 2^{2n} = 4^n$$

$$a = b$$

Sol.13 B

$$\text{Let } (2x^2 - 3x + 1) = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_{22}x^{22} \quad \dots (1)$$

$$\text{Put } x = -1$$

$$\Rightarrow a_0 - a_1 + a_2 - a_3 + \dots + a_{22}$$

$$= (2 + 3 + 1)^{11} = 6^{11} \quad \dots (2)$$

$$\text{Put } x = 1$$

$$\Rightarrow a_0 + a_1 + a_2 + a_3 + \dots + a_{22} = 0 \quad \dots (3)$$

adding (2) & (3)

$$\Rightarrow 2[a_0 + a_2 + a_4 + \dots + a_{22}] = 6^{11}$$

$$\Rightarrow a_0 + a_2 + a_4 + \dots + a_{22} = \frac{6^{11}}{2} = \frac{6 \cdot 6^{10}}{2} = 3 \cdot 6^{10}$$

Sol.14 C

$$\begin{aligned}
 {}^{18}C_{r-2} + 2 \cdot {}^{18}C_{r-1} + {}^{18}C_r &\geq {}^{20}C_{13} \\
 \Rightarrow {}^{18}C_{r-2} + {}^{18}C_{r-1} + {}^{18}C_{r-1} + {}^{18}C_r &\geq {}^{20}C_{13} \\
 \Rightarrow {}^{19}C_{r-1} + {}^{19}C_r &\geq {}^{20}C_{13} \Rightarrow {}^{20}C_r \geq {}^{20}C_{13}
 \end{aligned}$$

$$\therefore {}^{20}C_{10} > {}^{20}C_{11} > {}^{20}C_{12} > {}^{20}C_{13}$$

$$\& {}^{20}C_{10} > {}^{20}C_9 > {}^{20}C_8 > {}^{20}C_7$$

$$r = 7, 8, 9, 10, 11, 12, 13 \Rightarrow \text{Total 7 elements}$$

Sol.15 C

$$(2x + 5y)^{13} \text{ greatest form for } x = 10, y = 2$$

$$\frac{\frac{n+1}{\left|\frac{x}{y}\right|+1} - 1 \leq r \leq \frac{n+1}{\left|\frac{x}{y}\right|+1}}$$

$$\Rightarrow \frac{14}{\left|\frac{2x}{5y}\right|+1} - 1 \leq r \leq \frac{14}{\left|\frac{2x}{5y}\right|+1}$$

$$\Rightarrow \frac{14}{3} - 1 \leq r \leq \frac{14}{3} \Rightarrow \frac{11}{3} \leq r \leq \frac{14}{3}$$

$$\Rightarrow 3.66 \dots \leq r \leq 4.666 \Rightarrow r = 4$$

$$\Rightarrow T_5 = {}^{13}C_4 (20)^9 (10)^4$$

Sol.16 D

$$\left(x^k + \frac{1}{x^{2k}}\right)^{3n}, \quad n \in \mathbb{N} \quad \text{Independent of } x$$

$$T_{r+1} = {}^{3n}C_r (x^k)^{3n-r} \left(\frac{1}{x^{2k}}\right)^r$$

$$= {}^{3n}C_r x^{3nk - rk - 2kr} = {}^{3n}C_r x^{3k(n-r)}$$

$$\text{For Constant term} \Rightarrow 3k(n-r) = 0 \Rightarrow n = r$$

$$\therefore T_{r+1} = {}^{3n}C_n \text{ true for any real } k \text{ or } K \in \mathbb{R}$$

Sol.17 B

$$\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + \dots + \frac{C_{10}}{11}$$

$$\sum_{r=0}^{10} T_{r+1} = \sum_{r=0}^{10} \frac{{}^{10}C_r}{r+1}$$

$$\begin{aligned}
 &= \sum_{r=0}^{10} \frac{1}{(10+1)} \cdot \frac{10+1}{r+1} \cdot {}^{10}C_r = \frac{1}{11} \sum_{r=0}^{10} {}^{11}C_{r+1} \\
 &= \frac{1}{11} [{}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{11}] \\
 &= \frac{1}{11} [{}^{11}C_0 + {}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{11} - {}^{11}C_0] \\
 &= \frac{1}{11} [2^{11} - 1] = \frac{2^{11} - 1}{11}
 \end{aligned}$$

Sol.18

$$P + f = (5 + 2\sqrt{6})^n, P, n \in \mathbb{N}, 0 < f < 1$$

$$\text{Let } f' = (5 - 2\sqrt{6})^n = |\text{rational} - \text{irrational}|^n < 1$$

$$P + f + f' = 2 [\text{Integer}] = \text{even integer}$$

$$\therefore f + f' = 1 \Rightarrow (f - 1) = -f'$$

$$\text{Now } f^2 - f + Pf - P = f(f - 1) + P(f - 1)$$

$$= (f - 1)(P + f) = -f'(P + f)$$

$$= -(5 - 2\sqrt{6})^n (5 + 2\sqrt{6})^n = -[5^2 - 2^2 \cdot 6]^n$$

$$= -[25 - 24]^n = -1 = \text{negative Integer}$$

Sol.19 B

$$(x+2)^{n-1} + (x+2)^{n-2}(x+1) + (x+2)^{n-3}$$

$$(x+1)^2 + \dots + (x+1)^{n-1}$$

$$= a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1}$$

$$= a^{n-1} \left[1 + \left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2 + \dots + \left(\frac{b}{a}\right)^{n-1} \right]$$

$$= a^{n-1} \frac{1 \cdot \left[\left(\frac{b}{a}\right)^n - 1 \right]}{\frac{b}{a} - 1} = a^{n-1} \frac{a^n - b^n}{a - b} \cdot \frac{a}{a^n}$$

$$= \frac{(x+2)^n - (x+1)^n}{x+2 - x - 1} = (2+x)^n - (1+x)^n$$

$$T_{r+1} \text{ term is } {}^nC_r 2^{n-r} x^r - {}^nC_r x^r$$

$$\text{coeff of } x^r \text{ is } {}^nC_r (2^{n-r} - 1)$$

Sol.20 A

$$(1+x+x^2)^{25} = a_0 + a_1x + a_2x^2 + \dots + a_{50}x^{50}$$

$$\text{put } x = 1$$

$$a_0 + a_1 + a_2 + a_3 + \dots + a_{49} + a_{50} = 3^{25} \dots (i)$$

$$\text{put } x = -1$$

$$a_0 - a_1 + a_2 - a_3 + \dots + a_{49} + a_{50} = (1 - 1 + 1)^{25} \dots (ii)$$

$$\text{adding (i) \& (ii)}$$

$$2[a_0 + a_2 + a_4 + \dots + a_{50}] = 3^{25} + 1$$

$$a_0 + a_2 + a_4 + \dots + a_{50} = \frac{3^{25} + 1}{2} = \frac{(4-1)^{25} + 1}{2}$$

$$= {}^{25}C_0 4^{25} - {}^{25}C_1 4^{24} + {}^{25}C_2 4^{23} - \dots + {}^{25}C_{24} 4^1 - 1 + 1$$

$$= \frac{4[{}^{25}C_0 4^{24} - {}^{25}C_1 4^{23} + \dots + {}^{25}C_{24}]}{2}$$

Sol.21 D

$$\text{coef of } x^4 \text{ in } (1-x+2x^2)^{12}$$

$$= {}^{12}C_0 (1-x)^{12} (2x^2)^0 + {}^{12}C_1 (1-x)^{11} (2x^2)$$

$$+ {}^{12}C_2 (1-x)^{10} (2x^2)^2 + \text{above } x^4 \text{ powers}$$

$$\text{terms of } x^4$$

$$= {}^{12}C_0 \cdot {}^{12}C_4 (-x)^4 + {}^{12}C_1 {}^{11}C_2 (-x)^2 2x^2$$

$$+ {}^{12}C_2 {}^{10}C_0 4x^4$$

$$= {}^{12}C_4 + 12 \cdot {}^{11}C_2 \cdot 2 + {}^{12}C_2 \cdot 4$$

$$= {}^{12}C_4 + 2 \cdot 3 \cdot \frac{12}{3} {}^{11}C_2 + {}^{12}C_2 \cdot 4$$

$$= {}^{12}C_3 + {}^{12}C_2 + 3({}^{12}C_2 + {}^{12}C_3) + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$$

$$= {}^{12}C_3 + 3({}^{12}C_2 + {}^{12}C_3) + {}^{12}C_2 + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_4$$

$$= {}^{12}C_3 + 3^{13}C_3 + {}^{13}C_3 + {}^{13}C_4$$

$$= {}^{12}C_3 + 3^{13}C_3 + {}^{14}C_4$$

Sol.22 B

$$\sum_{k=1}^{n-r} {}^{n-k}C_r = {}^xC_y$$

$$\text{L.H.S.} = {}^{n-1}C_r + {}^{n-2}C_r + {}^{n-3}C_r + \dots + {}^rC_r$$

$$= {}^rC_r + {}^{r+1}C_r + \dots + {}^{n-2}C_r + {}^{n-1}C_r$$

$$\left\{ {}^rC_r = \frac{r+1}{r+1} {}^rC_r = {}^{r+1}C_{r+1} \right\}$$

$$= {}^{r+1}C_{r+1} + {}^{r+1}C_r + {}^{r+2}C_r + \dots + {}^{n-1}C_r$$

$$= {}^{r+2}C_{r+1} + {}^{r+2}C_r + \dots + {}^{n-1}C_r$$

$$= {}^{r+2}C_{r+1} + \dots + {}^{n-1}C_r$$

$$= {}^{n-1}C_{r+1} + {}^{n-1}C_r$$

$$= {}^nC_{r+1} = {}^xC_y \Rightarrow x = n, y = r + 1$$

Sol.23 BCoeff of α^t in

$$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 + \dots + (\alpha + q)^{m-1}$$

$$\because a \neq -q, \quad p \neq q$$

$$\text{Let } \alpha + p = x \quad \& \quad \alpha + q = y$$

$$= x^{m-1} + x^{m-2}y + x^{m-3}y^2 + \dots + y^{m-1}$$

$$= x^{m-1} \left[1 - \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 + \dots + \left(\frac{y}{x}\right)^{m-1} \right]$$

$$= x^{m-1} \frac{\left[1 - \left(\frac{y}{x}\right)^m \right]}{\left(1 - \frac{y}{x} \right)}$$

$$= \frac{x^{m-1}}{x^m} \frac{x^m - y^m}{x - y} \cdot x = \frac{(\alpha + p)^m - (\alpha + q)^m}{\alpha + p - \alpha - q}$$

$$= \frac{1}{(p - q)} [(\alpha + p)^m - (\alpha + q)^m]$$

$$= \text{coeff of } \alpha^t = \left(\frac{{}^m C_t p^{m-t} - {}^m C_t q^{m-t}}{p - q} \right)$$

Sol.24 B

$$(1 + x + x^2 + \dots + x^9)^{-1} \quad (|x| < 1)$$

$$= \left[\frac{1 \cdot (1 - x^{10})}{1 - x} \right]^{-1} = \frac{(1 - x)}{(1 - x^{10})} = (1 - x)(1 - x^{10})^{-1}$$

$$= (1 - x)[1 + (x^{10}) + (x^{10})^2 + (x^{10})^3 + \dots \infty]$$

$$= (1 - x)[1 + x^{10} + x^{20} + x^{30} + x^{40} + \dots + x^{400}$$

$$+ x^{410} + \dots \infty]$$

$$= \text{coeff of } x^{401} \text{ is } (-1)$$

Sol.25 C

$3^{1/3}$	$7^{1/7}$	1
0	0	10
3	0	7
6	0	4
9	0	1
3	7	0
0	7	3

\therefore no. of terms are 6

$$\text{Alter : } {}^{10}C_r \left(1 + 3^{\frac{1}{3}}\right)^{10-0} \Rightarrow {}^{10}C_r \left(1 + 3^{\frac{1}{3}}\right)^r$$

should be $r = 0, 3, 6, 9$

$$\left(1 + 3^{\frac{1}{3}}\right)^{10-7} \Rightarrow {}^{10}C_r \left(3^{\frac{1}{3}}\right)^r \text{ r should be } r = 0, 3$$

$$\text{corresponding } \left(7^{\frac{1}{7}}\right)^0 \left(3^{\frac{1}{3}}\right)^r$$

value of $r = 0, 3, 6, 9 = 4$ values

$$\text{corresponding } \left(7^{\frac{1}{7}}\right)^0 \left(3^{\frac{1}{3}}\right)^r$$

value of $r = 0, 3 = 2$ values

Total 6 values

Sol.26 CSum of the coeff of degree r is

$$(1 + x)^n (1 + y)^n (1 + z)^n$$

$$= \left(\sum_{k=0}^n {}^n C_k x^k \right) \left(\sum_{s=0}^n {}^n C_s y^s \right) \left(\sum_{t=0}^n {}^n C_t z^t \right)$$

$$= \sum_{0 \leq k, s, t \leq n} ({}^n C_k) ({}^n C_s) ({}^n C_t) x^k y^s z^t$$

$$\text{degree } m = k + s + t = r$$

$$\text{sum of coeff} = \sum_{k, s, t \geq 0} {}^n C_k \cdot {}^n C_s \cdot {}^n C_t$$

= the number of way of choosing a total number r balls out of n white, n block and n red balls.

$$= {}^{3n}C_r$$